

Pari-GP reference card

(PARI-GP version 2.11.0)

Note: optional arguments are surrounded by braces {}.

To start the calculator, type its name in the terminal: **gp**

To exit **gp**, type **quit**, **\q**, or **<C-D>** at prompt.

Help

describe function	?function
extended description	??keyword
list of relevant help topics	???pattern
name of GP-1.39 function f in GP-2.*	whatnow(f)

Input/Output

previous result, the result before	%, %', %'', etc.
n -th result since startup	% n
separate multiple statements on line	;
extend statement on additional lines	\
extend statements on several lines	{seq ₁ ; seq ₂ ;}
comment	/* ... */
one-line comment, rest of line ignored	\\ ...

Metacommands & Defaults

set default d to val	default({ d },{ val })
toggle timer on/off	#
print time for last result	##
print defaults	\d
set debug level to n	\g n
set memory debug level to n	\gm n
set n significant digits / bits	\p n , \pb n
set n terms in series	\ps n
quit GP	\q
print the list of PARI types	\t
print the list of user-defined functions	\u
read file into GP	\r <i>filename</i>

Debugger / break loop

get out of break loop	break or <C-D>
go up/down n frames	dbg_up({ n }), dbg_down
set break point	breakpoint()
examine object o	dbg_x(o)
current error data	dbg_err()
number of objects on heap and their size	getheap()
total size of objects on PARI stack	getstack()

PARI Types & Input Formats

t_INT . Integers; hex, binary	± 31 ; $\pm 0x1F$, $\pm 0b101$
t_REAL . Reals	± 3.14 , 6.022 E23
t_INTMOD . Integers modulo m	Mod(n , m)
t_FRAC . Rational Numbers	n/m
t_FFELT . Elt in finite field \mathbf{F}_q	ffgen(q , 't)
t_COMPLEX . Complex Numbers	$x + y * I$
t_PADIC . p -adic Numbers	$x + O(p^k)$
t_QUAD . Quadratic Numbers	$x + y * \text{quadgen}(D, 'w)$
t_POLMOD . Polynomials modulo g	Mod(f , g)
t_POL . Polynomials	$a * x^n + \dots + b$
t_SER . Power Series	$f + O(x^k)$
t_RFRAC . Rational Functions	f/g
t_QFI / t_QFR . Imag/Real binary quad. form	Qfb($a, b, c, \{d\}$)
t_VEC / t_COL . Row/Column Vectors	$[x, y, z]$, $[x, y, z] \sim$
t_VEC integer range	$[1..10]$

t_VECSMALL . Vector of small ints	Vecsmall($[x, y, z]$)
t_MAT . Matrices	$[a, b; c, d]$
t_LIST . Lists	List($[x, y, z]$)
t_STR . Strings	"abc"
t_INFINITY . $\pm\infty$	+oo, -oo

Reserved Variable Names

$\pi = 3.14\dots$, $\gamma = 0.57\dots$, $C = 0.91\dots$	Pi, Euler, Catalan
square root of -1	I
Landau's big-oh notation	O

Information about an Object

PARI type of object x	type(x)
length of x / size of x in memory	# x , sizebyte(x)
real precision / bit precision of x	precision(x), bitprecision
p -adic, series prec. of x	padicprec(x), serprec

Operators

basic operations	+, -, *, /, ^, sqr
$i=i+1$, $i=i-1$, $i=i*j$, ...	i++, i--, i*=j,...
eulidean quotient, remainder	$x \backslash y$, $x \backslash y$, $x \backslash y$, divrem(x, y)
shift x left or right n bits	$x << n$, $x >> n$ or shift($x, \pm n$)
multiply by 2^n	shiftmul(x, n)
comparison operators	<=, <, >=, >, ==, !=, ==, lex, cmp
boolean operators (or, and, not)	, &&, !
bit operations	bitand, bitneg, bitor, bitxor, bitneginv
maximum/minimum of x and y	max, min(x, y)
sign of $x = -1, 0, 1$	sign(x)
binary exponent of x	exponent(x)
derivative of f	f'
differential operator	diffop($f, v, d, \{n = 1\}$)
quote operator (formal variable)	'x
assignment	x = <i>value</i>
simultaneous assignment $x \leftarrow v_1, y \leftarrow v_2$	[x,y] = v

Select Components

n -th component of x	component(x, n)
n -th component of vector/list x	$x[n]$
components $a, a + 1, \dots, b$ of vector x	$x[a..b]$
(m, n) -th component of matrix x	$x[m, n]$
row m or column n of matrix x	$x[m,]$, $x[, n]$
numerator/denominator of x	numerator(x), denominator

Random Numbers

random integer/prime in $[0, N[$	random(N), randomprime
get/set random seed	getrand, setrand(s)

Conversions

to vector, matrix, vec. of small ints	Col/Vec, Mat, Vecsmall
to list, set, map, string	List, Set, Map, Str
create PARI object ($x \bmod y$)	Mod(x, y)
make x a polynomial of v	Pol($x, \{v\}$)
as Pol, etc., starting with constant term	Polrev, Vecrev, Colrev
make x a power series of v	Ser($x, \{v\}$)
string from bytes / from format+args	Strchr, Strprintf
TeX string	Strtex(x)
convert x to simplest possible type	simplify(x)
object x with real precision n	precision(x, n)
object x with bit precision n	bitprecision(x, n)
set precision to p digits in dynamic scope	localprec(p)
set precision to p bits in dynamic scope	localbitprec(p)

Conjugates and Lifts

conjugate of a number x	conj(x)
norm of x , product with conjugate	norm(x)
L^p norm of x (L^∞ if no p)	normlp($x, \{p\}$)
square of L^2 norm of x	norml2(x)
lift of x from Mods and p -adics	lift, centerlift(x)
recursive lift	liftall
lift all t_INT and t_PADIC (\rightarrow t_INT)	liftint
lift all t_POLMOD (\rightarrow t_POL)	lifttpol

Lists, Sets & Maps

Sets (= row vector with strictly increasing entries w.r.t. **cmp**)

intersection of sets x and y	setintersect(x, y)
set of elements in x not belonging to y	setminus(x, y)
union of sets x and y	setunion(x, y)
does y belong to the set x	setsearch($x, y, \{flag\}$)
set of all $f(x, y)$, $x \in X$, $y \in Y$	setbinop(f, X, Y)
is x a set ?	setisset(x)

Lists. create empty list: $L = \text{List}()$

append x to list L	listput($L, x, \{i\}$)
remove i -th component from list L	listpop($L, \{i\}$)
insert x in list L at position i	listinsert(L, x, i)
sort the list L in place	listsort($L, \{flag\}$)

Maps. create empty dictionary: $M = \text{Map}()$

attach value v to key k	mapput(M, k, v)
recover value attach to key k or error	mapget(M, k)
is key k in the dict ? (set v to $M(k)$)	mapisdefined($M, k, \{\&v\}$)
remove k from map domain	mapdelete(M, k)

GP Programming

User functions and closures

x, y are formal parameters; y defaults to Pi if parameter omitted;
 z, t are local variables (lexical scope), z initialized to 1.

```
fun(x, y=Pi) = my(z=1, t); seq
fun = (x, y=Pi) -> my(z=1, t); seq
```

attach a help message to f	addhelp(f)
undefine symbol s (also kills help)	kill(s)
Control Statements (X : formal parameter in expression seq)	
if $a \neq 0$, evaluate seq_1 , else seq_2	if($a, \{seq_1\}, \{seq_2\}$)

eval. seq for $a \leq X \leq b$	for($X = a, b, seq$)
...for primes $a \leq X \leq b$	forprime($X = a, b, seq$)
...for primes $\equiv a \pmod{q}$	forprimestep($X = a, b, q, seq$)
...for composites $a \leq X \leq b$	forcomposite($X = a, b, seq$)
...for $a \leq X \leq b$ stepping s	forstep($X = a, b, s, seq$)
...for X dividing n	fordiv(n, X, seq)
... $X = [n, factor(n)]$, $a \leq n \leq b$	forfactored($X = a, b, seq$)
...as above, n squarefree	forsquarefree($X = a, b, seq$)
... $X = [d, factor(d)]$, $d \mid n$	fordivfactored(n, X, seq)
multivariable for , lex ordering	forvec($X = v, seq$)
loop over partitions of n	forpart($p = n, seq$)
...permutations of S	forperm(S, p, seq)
...subsets of $\{1, \dots, n\}$	forsubset(n, p, seq)
... k -subsets of $\{1, \dots, n\}$	forsubset($[n, k], p, seq$)
...vectors v , $q(v) \leq B$; $q > 0$	forqfvec(v, q, b, seq)
... $H < G$ finite abelian group	forsubgroup($H = G$)

evaluate seq until $a \neq 0$	until(a, seq)
while $a \neq 0$, evaluate seq	while(a, seq)
exit n innermost enclosing loops	break({ n })
start new iteration of n -th enclosing loop	next({ n })
return x from current subroutine	return({ x })

Exceptions, warnings

raise an exception / warn

type of error message E

try seq_1 , evaluate seq_2 on error

Functions with closure arguments / results

select from v according to f

apply f to all entries in v

evaluate $f(a_1, \dots, a_n)$

evaluate $f(\dots f(f(a_1, a_2), a_3) \dots, a_n)$

calling function as closure

Sums & Products

sum $X = a$ to $X = b$, initialized at x

sum entries of vector v

product of all vector entries

sum $expr$ over divisors of n

... assuming $expr$ multiplicative

product $a \leq X \leq b$, initialized at x

product over primes $a \leq X \leq b$

Sorting

sort x by k -th component

min. m of x ($m = x[i]$), max.

does y belong to x , sorted wrt. f

Input/Output

print with/without $\backslash n$, \TeX format

pretty print matrix

print fields with separator

formatted printing

write $args$ to file

write x in binary format

read file into GP

... return as vector of lines

... return as vector of strings

read a string from keyboard

Files and file descriptors

File descriptors allows efficient small consecutive reads or writes from or to a given file. The argument n below is always a descriptor, attached to a file in **r**(ead), **w**(rite) or **a**(ppend) mode.

get descriptor n for file $path$ in given $mode$
... from shell cmd output (pipe)

close descriptor

commit pending write operations

read logical line from file

... raw line from file

write $s \backslash n$ to file

... write s to file

Timers

CPU time in ms and reset timer

CPU time in ms since gp startup

time in ms since UNIX Epoch

timeout command after s seconds

Interface with system

allocates a new stack of s bytes

alias old to new

install function from library

execute system command a

... and feed result to GP

... returning GP string

error(), **warning()**

errname() (E)

iferr() (seq_1, E, seq_2)

select() (f, v)

apply() (f, v)

call() (f, a)

fold() (f, a)

self()

sum() ($X = a, b, expr, \{x\}$)

vecsum() (v)

vecprod() (v)

sumdiv() ($n, X, expr$)

sumdivmult() ($n, X, expr$)

prod() ($X = a, b, expr, \{x\}$)

prodeuler() ($X = a, b, expr$)

vecsrt() ($x, \{k\}, \{fl = 0\}$)

vecmin() ($x, \{\&i\}$), **vecmax**

vecsearch() ($x, y, \{f\}$)

print(), **print1()**, **printtex**

printp

printsep() (sep, \dots), **printsep1**

printf()

write(), **writel()**, **writetex()** ($file, args$)

writebin() ($file, x$)

read() ($\{file\}$)

readvec() ($\{file\}$)

readstr() ($\{file\}$)

input()

fileopen() ($path, mode$)

fileextern() (cmd)

fileclose() (n)

fileflush() (n)

fileread() (n)

filereadstr() (n)

filewrite() (n, s)

filewritel() (n, s)

gettime()

getabstime()

getwalltime()

alarm() ($s, expr$)

allocatemem() ($\{s\}$)

alias() (new, old)

install() ($f, code, \{gpf\}, \{lib\}$)

system() (a)

extern() (a)

externstr() (a)

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get $\$VAR$ from environment

expand env. variable in string

getenv() ("VAR")

Strexpend() (x)

Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables and must be free of side effects. Enabled if threading engine is not *single* in gp header.

evaluate f on $x[1], \dots, x[n]$

evaluate closures $f[1], \dots, f[n]$

as **select**

as **sum**

as **vector**

eval f for $i = a, \dots, b$

... for p prime in $[a, b]$

... multivariate

declare x as inline (allows to use as global)

stop inlining

parapply() (f, x)

pareval() (f)

parselect() ($f, A, \{flag\}$)

parsum() ($i = a, b, expr, \{x\}$)

parvector() ($n, i, \{expr\}$)

parfor() ($i = a, \{b\}, f, \{r\}, \{f_2\}$)

parforprime() ($p = a, \{b\}, f, \{r\}, \{f_2\}$)

parforvec() ($X = v, f, \{r\}, \{f_2\}, \{flag\}$)

inline() (x)

uninline()

Linear Algebra

dimensions of matrix x

multiply two matrices

... assuming result is diagonal

concatenation of x and y

extract components of x

transpose of vector or matrix x

adjoint of the matrix x

eigenvectors/values of matrix x

characteristic/minimal polynomial of x

trace/determinant of matrix x

permanent of matrix x

Frobenius form of x

QR decomposition

apply **matqr**'s transform to v

Constructors & Special Matrices

$\{g(x): x \in v \text{ s.t. } f(x)\}$

$\{x: x \in v \text{ s.t. } f(x)\}$

$\{g(x): x \in v\}$

row vec. of $expr$ eval'ed at $1 \leq i \leq n$

col. vec. of $expr$ eval'ed at $1 \leq i \leq n$

vector of small ints

$[c, c \cdot x, \dots, c \cdot x^n]$

matrix $1 \leq i \leq m, 1 \leq j \leq n$

define matrix by blocks

diagonal matrix with diagonal x

is x diagonal?

$x \cdot \text{matdiagonal}(d)$

$n \times n$ identity matrix

Hessenberg form of square matrix x

$n \times n$ Hilbert matrix $H_{ij} = (i + j - 1)^{-1}$

$n \times n$ Pascal triangle

companion matrix to polynomial x

Sylvester matrix of x

matsize() (x)

$x * y$

matmultodiagonal() (x, y)

concat() ($x, \{y\}$)

vecextract() ($x, y, \{z\}$)

mattranspose() (x) or $x \sim$

matadjoint() (x)

mateigen() (x)

charpoly() (x), **minpoly**

trace() (x), **matdet**

matpermanent() (x)

matfrobenius() (x)

matqr() (x)

mathouseholder() (Q, v)

[g(x) | x <- v, f(x)]

[x | x <- v, f(x)]

[g(x) | x <- v]

vector() ($n, \{i\}, \{expr\}$)

vectorv() ($n, \{i\}, \{expr\}$)

vectorsmall() ($n, \{i\}, \{expr\}$)

powers() ($x, n, \{c = 1\}$)

matrix() ($m, n, \{i\}, \{j\}, \{expr\}$)

matconcat() (B)

matdiagonal() (x)

matisdiagonal() (x)

matmuldiagonal() (x, d)

matid() (n)

mathess() (x)

mathilbert() (n)

matpascal() ($n - 1$)

matcompanion() (x)

polsylvestermatrix() (x)

Gaussian elimination

kernel of matrix x

intersection of column spaces of x and y

solve $MX = B$ (M invertible)

one sol of $M * X = B$

basis for image of matrix x

columns of x *not* in **matimage**

supplement columns of x to get basis

rows, cols to extract invertible matrix

rank of the matrix x

solve $MX = B \bmod D$

image mod D

kernel mod D

inverse mod D

determinant mod D

matker() ($x, \{flag\}$)

matintersect() (x, y)

matsolve() (M, B)

matinverseimage() (M, B)

matimage() (x)

matimagecompl() (x)

mat supplement() (x)

matindexrank() (x)

matrank() (x)

mat solvemod() (M, D, B)

matimagemod() (M, D)

matkermod() (M, D)

matinvmod() (M, D)

matdetmod() (M, D)

Lattices & Quadratic Forms

Quadratic forms

evaluate ${}^t x Q y$

evaluate ${}^t x Q x$

signature of quad form ${}^t y * x * y$

decomp into squares of ${}^t y * x * y$

eigenvalues/vectors for real symmetric x

qfeval() ($\{Q = id\}, x, y$)

qfeval() ($\{Q = id\}, x$)

qfsign() (x)

qfgaussred() (x)

qfjacobi() (x)

HNF and SNF

upper triangular Hermite Normal Form

HNF of x where d is a multiple of $\det(x)$

multiple of $\det(x)$

HNF of $(x \mid \text{diagonal}(D))$

elementary divisors of x

elementary divisors of $\mathbf{Z}[a]/(f'(a))$

integer kernel of x

Z-module \leftrightarrow **Q**-vector space

mathnf() (x)

mathnfmod() (x, d)

matdetint() (x)

mathnfmodid() (x, D)

mat snf() (x)

poldiscreduced() (f)

matkerint() (x)

matrixqz() (x, p)

Lattices

LLL-algorithm applied to columns of x

... for Gram matrix of lattice

find up to m sols of **qfnorm** (x, y) $\leq b$

$v, v[i] :=$ number of y s.t. **qfnorm** (x, y) = i

perfection rank of x

find isomorphism between q and Q

precompute for isomorphism test with q

automorphism group of q

convert **qfauto** for GAP/Magma

orbits of V under $G \subset \text{GL}(V)$

qflll() ($x, \{flag\}</$

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Coefficients, variables and basic operators

degree of f	<code>poldegree(f)</code>
coef. of degree n of f , leading coef.	<code>polcoef(f,n), pollead</code>
main variable / all variables in f	<code>variable(f), variables(f)</code>
replace x by y in f	<code>subst(f,x,y)</code>
evaluate f replacing vars by their value	<code>eval(f)</code>
replace polynomial expr. $T(x)$ by y in f	<code>substpol(f,T,y)</code>
replace x_1, \dots, x_n by y_1, \dots, y_n in f	<code>substvec(f,x,y)</code>
reciprocal polynomial $x^{\deg f} f(1/x)$	<code>polrecip(f)</code>
gcd of coefficients of f	<code>content(f)</code>
derivative of f w.r.t. x	<code>deriv(f,{x})</code>
formal integral of f w.r.t. x	<code>intformal(f,{x})</code>
formal sum of f w.r.t. x	<code>sumformal(f,{x})</code>

Constructors & Special Polynomials

interpolating pol. eval. at a	<code>polinterpolate(X,{Y},{a})</code>
$P_n, T_n/U_n, H_n$	<code>pollegendre, polchebyshev, polhermite</code>
n -th cyclotomic polynomial Φ_n	<code>polcyclo(n,{v})</code>
return n if $f = \Phi_n$, else 0	<code>poliscyclo(f)</code>
is f a product of cyclotomic polynomials?	<code>poliscycloprod(f)</code>
Zagier's polynomial of index (n,m)	<code>polzagier(n,m)</code>

Resultant, elimination

discriminant of polynomial f	<code>poldisc(f)</code>
find factors of <code>poldisc(f)</code>	<code>poldiscfactors(f)</code>
resultant $R = \text{Res}_v(f,g)$	<code>polresultant(f,g,{v})</code>
$[u,v,R], xu + yv = \text{Res}_v(f,g)$	<code>polresultantext(x,y,{v})</code>
solve Thue equation $f(x,y) = a$	<code>thue(t,a,{sol})</code>
initialize t for Thue equation solver	<code>thueinit(f)</code>

Roots and Factorization (Complex/Real)

complex roots of f	<code>polroots(f)</code>
bound complex roots of f	<code>polrootsbound(f)</code>
number of real roots of f (in $[a,b]$)	<code>polsturm(f,{{a,b}})</code>
real roots of f (in $[a,b]$)	<code>polrootsreal(f,{{a,b}})</code>
complex embeddings of <code>t_POLMOD z</code>	<code>conjvec(z)</code>

Roots and Factorization (Finite fields)

factor $f \bmod p$, roots	<code>factormod(f,p), polrootsmod</code>
factor f over $\mathbf{F}_p[x]/(T)$, roots	<code>factormod(f,[T,p]), polrootsmod</code>
squarefree factorization of f in $\mathbf{F}_q[x]$	<code>factormodSQF(f,{D})</code>
distinct degree factorization of f in $\mathbf{F}_q[x]$	<code>factormodDDF(f,{D})</code>

Roots and Factorization (p -adic fields)

factor f over \mathbf{Q}_p , roots	<code>factorpadic(f,p,r), polrootspadic</code>
p -adic root of f congruent to $a \bmod p$	<code>padicappr(f,a)</code>
Newton polygon of f for prime p	<code>newtonpoly(f,p)</code>
Hensel lift $A/\text{lc}(A) = \prod_i B[i] \bmod p^e$	<code>polhensellift(A,B,p,e)</code>
extensions of \mathbf{Q}_p of degree N	<code>padicfields(p,N)</code>

Roots and Factorization (Miscellaneous)

symmetric powers of roots of f up to n	<code>polsym(f,n)</code>
Graeffe transform of f , $g(x^2) = f(x)f(-x)$	<code>polgraeffe(f)</code>
factor f over coefficient field	<code>factor(f)</code>
cyclotomic factors of $f \in \mathbf{Q}[X]$	<code>polcyclofactors(f)</code>

Finite Fields

A finite field is encoded by any element (`t_FFELT`).

find irreducible $T \in \mathbf{F}_p[x]$, $\deg T = n$	<code>ffinit(p,n,{x})</code>
Create t in $\mathbf{F}_q \simeq \mathbf{F}_p[t]/(T)$	<code>t = ffgen(T,'t)</code>
... indirectly, with implicit T	<code>t = ffgen(q,'t); T = t.mod</code>
map M from $\mathbf{F}_q \ni a$ to $\mathbf{F}_{q^k} \ni b$	<code>m = ffmbed(a,b)</code>
build K from $\mathbf{F}_q[x]/(P)$ extending $\mathbf{F}_q \ni a$,	<code>ffextend(a,P)</code>
evaluate map m on x	<code>ffmap(m,x)</code>
inverse map of m	<code>ffinvmap(m)</code>
compose maps $m \circ n$	<code>ffcompomap(m,n)</code>
F^n over $\mathbf{F}_q \ni a$	<code>fffrobenius(a,n)</code>
$\#\{\text{monic irred. } T \in \mathbf{F}_q[x], \deg T = n\}$	<code>ffnbirred(q,n)</code>

Formal & p -adic Series

truncate power series or p -adic number	<code>truncate(x)</code>
valuation of x at p	<code>valuation(x,p)</code>
Dirichlet and Power Series	
Taylor expansion around 0 of f w.r.t. x	<code>taylor(f,x)</code>
Laurent series expansion around 0 up to x^k	<code>laurentseries(f,k)</code>
$\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$	<code>serconvol(a,b)</code>
$f = \sum a_k t^k$ from $\sum (a_k/k!) t^k$	<code>serlaplace(f)</code>
reverse power series F so $F(f(x)) = x$	<code>serreverse(f)</code>
remove terms of degree $< n$ in f	<code>serchop(f,n)</code>
Dirichlet series multiplication / division	<code>dirmul, dirdiv(x,y)</code>
Dirichlet Euler product (b terms)	<code>direuler(p=a,b,expr)</code>

Transcendental and p -adic Functions

real, imaginary part of x	<code>real(x), imag(x)</code>
absolute value, argument of x	<code>abs(x), arg(x)</code>
square/nth root of x	<code>sqrt(x), sqrtn(x,n,{&z})</code>
trig functions	<code>sin, cos, tan, cotan, sinc</code>
inverse trig functions	<code>asin, acos, atan</code>
hyperbolic functions	<code>sinh, cosh, tanh, cotanh</code>
inverse hyperbolic functions	<code>asinh, acosh, atanh</code>
$\log(x), \log(1+x), e^x, e^x - 1$	<code>log, log1p, exp, expm1</code>
Euler Γ function, $\log \Gamma, \Gamma'/\Gamma$	<code>gamma, lngamma, psi</code>
half-integer gamma function $\Gamma(n+1/2)$	<code>gammah(n)</code>
Riemann's zeta $\zeta(s) = \sum n^{-s}$	<code>zeta(s)</code>
Hurwitz's $\zeta(s,x) = \sum (n+x)^{-s}$	<code>zetahurwitz(s,x)</code>
multiple zeta value (MZV), $\zeta(s_1, \dots, s_k)$	<code>zetamult(s,{T})</code>
... init T for MZV with $s_1 + \dots + s_k \leq w$	<code>zetamultinit(w)</code>
all MZVs for all weights $\sum s_i \leq n$	<code>zetamultall(n)</code>
convert MZV id to $[s_1, \dots, s_k]$	<code>zetamultconvert(f,{flag})</code>
incomplete Γ function ($y = \Gamma(s)$)	<code>incgam(s,x,{y})</code>
complementary incomplete Γ	<code>incgamc(s,x)</code>
$\int_x^\infty e^{-t} dt/t, (2/\sqrt{\pi}) \int_x^\infty e^{-t^2} dt$	<code>eint1, erfc</code>
dilogarithm of x	<code>dilog(x)</code>
m -th polylogarithm of x	<code>polylog(m,x,{flag})</code>
U -confluent hypergeometric function	<code>hyperu(a,b,u)</code>
Bessel $J_n(x), J_{n+1/2}(x)$	<code>besselj(n,x), besseljh(n,x)</code>
Bessel $I_\nu, K_\nu, H_\nu^1, H_\nu^2, N_\nu$	<code>(bessel)i,k,h1,h2,n</code>
Lambert $W: x$ s.t. $xe^x = y$	<code>lambertw(y)</code>
Teichmuller character of p -adic x	<code>teichmuller(x)</code>

Iterations, Sums & Products

Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential (DE) methods: either a scalar ($a \in \mathbf{C}$, regular) or $\pm\infty$ (decreasing at least as x^{-2}) or

$(x-a)^{-\alpha}$ singularity	<code>[a,a]</code>
exponential decrease $e^{-\alpha x }$	<code>[$\pm\infty, \alpha$], $\alpha > 0$</code>
slow decrease $ x ^\alpha$	<code>... $\alpha < -1$</code>
oscillating as $\cos(kx)$	<code>$\alpha = k\mathbf{I}, k > 0$</code>
oscillating as $\sin(kx)$	<code>$\alpha = -k\mathbf{I}, k > 0$</code>
numerical integration	<code>intnum($x=a,b,f,{T}$)</code>
weights T for intnum	<code>intnuminit($a,b,K,{m}$)</code>
weights T incl. kernel K	<code>intfuncinit($a,b,K,{m}$)</code>
integrate $(2i\pi)^{-1} f$ on circle $ z-a = R$	<code>intcirc($x=a,R,f,{T}$)</code>

Other integration methods

n -point Gauss-Legendre	<code>intnumgauss($x=a,b,f,{n}$)</code>
weights for n -point Gauss-Legendre	<code>intnumgaussinit({n})</code>
Romberg integration (low accuracy)	<code>intnumromb($x=a,b,f,{flag}$)</code>

Numerical summation

sum of series $f(n), n \geq a$ (low accuracy)	<code>suminf($n=a,expr$)</code>
sum of alternating/positive series	<code>sumalt, sumpos</code>
sum of series using Euler-Maclaurin	<code>sumnum($n=a,f,{T}$)</code>
$\sum_{n \geq a} F(n)$, F rational function	<code>sumnumrat(F,a)</code>
$\dots \sum_{n \geq a} (-1)^n F(n)$	<code>sumaltrat(F,a)</code>
$\dots \sum_{p \geq a} F(p^s)$	<code>sumeulerrat($F,{s=1},{a=2}$)</code>
weights for <code>sumnum</code> , a as in DE	<code>sumnuminit({∞,a})</code>
sum of series by Monien summation	<code>sumnummonien($n=a,f,{T}$)</code>
weights for <code>sumnummonien</code>	<code>sumnummonieninit({∞,a})</code>
sum of series using Abel-Plana	<code>sumnumap($n=a,f,{T}$)</code>
weights for <code>sumnumap</code> , a as in DE	<code>sumnumapinit({∞,a})</code>
sum of series using Lagrange	<code>sumnumlagrange($n=a,f,{T}$)</code>
weights for <code>sumnumlagrange</code>	<code>sumnumlagrangeinit</code>

Products

product $a \leq X \leq b$, initialized at x	<code>prod($X=a,b,expr,{x}$)</code>
product over primes $a \leq X \leq b$	<code>prodeuler($X=a,b,expr$)</code>
infinite product $a \leq X \leq \infty$	<code>prodinfn($X=a,expr$)</code>
$\prod_{n \geq a} F(n)$, F rational function	<code>prodnumrat(F,a)</code>
$\dots \prod_{p \geq a} F(p^s)$	<code>prodeulerrat($F,{s=1},{a=2}$)</code>

Other numerical methods

real root of f in $[a,b]$; bracketed root	<code>solve($X=a,b,f$)</code>
... by interval splitting	<code>solvestep($X=a,b,f,{flag=0}$)</code>
limit of $f(t), t \rightarrow \infty$	<code>limitnum(f,{k},{alpha})</code>
asymptotic expansion of f at ∞	<code>asypnum(f,{k},{alpha})</code>
numerical derivation w.r.t $x: f'(a)$	<code>derivnum($x=a,f$)</code>
evaluate continued fraction F at t	<code>contfraceval($F,t,{L}$)</code>
power series to cont. fraction (L terms)	<code>contfracinit($S,{L}$)</code>
Padé approximant (deg. denom. $\leq B$)	<code>bestapprPade($S,{B}$)</code>

Elementary Arithmetic Functions

vector of binary digits of $ x $	<code>binary(x)</code>
bit number n of integer x	<code>bittest(x, n)</code>
Hamming weight of integer x	<code>hammingweight(x)</code>
digits of integer x in base B	<code>digits($x, \{B = 10\}$)</code>
sum of digits of integer x in base B	<code>sumdigits($x, \{B = 10\}$)</code>
integer from digits	<code>fromdigits($v, \{B = 10\}$)</code>
ceiling/floor/fractional part	<code>ceil, floor, frac</code>
round x to nearest integer	<code>round($x, \{\&e\}$)</code>
truncate x	<code>truncate($x, \{\&e\}$)</code>
gcd/LCM of x and y	<code>gcd(x, y), lcm(x, y)</code>
gcd of entries of a vector/matrix	<code>content(x)</code>
Primes and Factorization	
extra prime table	<code>addprimes()</code>
add primes in v to prime table	<code>addprimes(v)</code>
remove primes from prime table	<code>removeprimes(v)</code>
Chebyshev $\pi(x)$, n -th prime p_n	<code>primepi(x), prime(n)</code>
vector of first n primes	<code>primes(n)</code>
smallest prime $\geq x$	<code>nextprime(x)</code>
largest prime $\leq x$	<code>preprime(x)</code>
factorization of x	<code>factor($x, \{lim\}$)</code>
...selecting specific algorithms	<code>factorint($x, \{flag = 0\}$)</code>
$n = df^2$, d squarefree/fundamental	<code>core($n, \{fl\}$), coredisc</code>
certificate for (prime) N	<code>primecert(N)</code>
verifies a certificate c	<code>primecertisvalid(c)</code>
convert certificate to Magma/PRIMO	<code>primecertexport</code>
recover x from its factorization	<code>factorback($f, \{e\}$)</code>
$x \in \mathbf{Z}$, $ x \leq X$, $\gcd(N, P(x)) \geq N$	<code>zncoppersmith($P, N, X, \{B\}$)</code>
divisors of N in residue class $r \bmod s$	<code>divisorslenstra(N, r, s)</code>
Divisors and multiplicative functions	
number of prime divisors $\omega(n)$ / $\Omega(n)$	<code>omega(n), bigomega</code>
divisors of n / number of divisors $\tau(n)$	<code>divisors(n), numdiv</code>
sum of (k -th powers of) divisors of n	<code>sigma($n, \{k\}$)</code>
Möbius μ -function	<code>moebius(x)</code>
Ramanujan's τ -function	<code>ramanujantau(x)</code>
Combinatorics	
factorial of x	<code>$x!$ or factorial(x)</code>
binomial coefficient $\binom{x}{k}$	<code>binomial($x, \{k\}$)</code>
Bernoulli number B_n as real/rational	<code>bernreal(n), bernfrac</code>
Bernoulli polynomial $B_n(x)$	<code>bernpol($n, \{x\}$)</code>
n -th Fibonacci number	<code>fibonacci(n)</code>
Stirling numbers $s(n, k)$ and $S(n, k)$	<code>stirling($n, k, \{flag\}$)</code>
number of partitions of n	<code>numbpart(n)</code>
k -th permutation on n letters	<code>numtoperm(n, k)</code>
convert permutation to (n, k) form	<code>permtotnum(v)</code>
order of permutation p	<code>permorder(p)</code>
signature of permutation p	<code>permsign(p)</code>
Multiplicative groups $(\mathbf{Z}/N\mathbf{Z})^*$, \mathbf{F}_q^*	
Euler ϕ -function	<code>eulerphi(x)</code>
multiplicative order of x (divides ϕ)	<code>znorder($x, \{o\}$), fforder</code>
primitive root mod q / x .mod	<code>znprimroot(q), ffprimroot(x)</code>
structure of $(\mathbf{Z}/n\mathbf{Z})^*$	<code>znstar(n)</code>
discrete logarithm of x in base g	<code>znlog($x, g, \{o\}$), fflag</code>
Kronecker-Legendre symbol $(\frac{x}{y})$	<code>kronecker(x, y)</code>
quadratic Hilbert symbol (at p)	<code>hilbert($x, y, \{p\}$)</code>

Miscellaneous	
integer square / n -th root of x	<code>sqrntint(x), sqrtnint(x, n)</code>
largest integer e s.t. $b^e \leq b$, $e = \lfloor \log_b(x) \rfloor$	<code>logint($x, b, \{\&z\}$)</code>
CRT: solve $z \equiv x$ and $z \equiv y$	<code>chinese(x, y)</code>
minimal u, v so $xu + yv = \gcd(x, y)$	<code>gcdext(x, y)</code>
continued fraction of x	<code>contfrac($x, \{b\}, \{lmax\}$)</code>
last convergent of continued fraction x	<code>confracpnqn(x)</code>
rational approximation to x (den. $\leq B$)	<code>bestappr($x, \{B\}k$)</code>
recognize $x \in \mathbf{C}$ as polmod mod $T \in \mathbf{Z}[X]$	<code>bestapprnf(x, T)</code>

Characters

Let $cyc = [d_1, \dots, d_k]$ represent an abelian group $G = \oplus (\mathbf{Z}/d_j\mathbf{Z}) \cdot g_j$ or any structure G affording a .cyc method; e.g. <code>znstar($q, 1$)</code> for Dirichlet characters. A character χ is coded by $[c_1, \dots, c_k]$ such that $\chi(g_j) = e(n_j/d_j)$.	
$\chi \cdot \psi$; χ^{-1} ; $\chi \cdot \psi^{-1}$; χ^k	<code>charmul, charconj, chardiv,, charpow</code>
order of χ	<code>charorder(cyc, χ)</code>
kernel of χ	<code>charker(cyc, χ)</code>
$\chi(x)$, G a GP group structure	<code>chareval($G, \chi, x, \{z\}$)</code>
Galois orbits of characters	<code>chargalois(G)</code>

Dirichlet Characters

initialize $G = (\mathbf{Z}/q\mathbf{Z})^*$	<code>G = znstar($q, 1$)</code>
convert datum D to $[G, \chi]$	<code>znchar(D)</code>
is χ odd?	<code>zncharisodd(G, χ)</code>
real $\chi \rightarrow$ Kronecker symbol $(D/.)$	<code>znchartokronecker(G, χ)</code>
conductor of χ	<code>zncharconductor(G, χ)</code>
$[G_0, \chi_0]$ primitive attached to χ	<code>znchartoprimitive(G, χ)</code>
induce $\chi \in \hat{G}$ to $\mathbf{Z}/N\mathbf{Z}$	<code>zncharinduce(G, χ, N)</code>
χ_p	<code>znchardecompose(G, χ, p)</code>
$\prod_p (\chi, N) \chi_p$	<code>znchardecompose(G, χ, Q)</code>
complex Gauss sum $G_a(\chi)$	<code>znchargauss(G, χ)</code>

Conrey labelling

Conrey label $m \in (\mathbf{Z}/q\mathbf{Z})^* \rightarrow$ character	<code>znconreychar(G, m)</code>
character \rightarrow Conrey label	<code>znconreyexp(G, χ)</code>
log on Conrey generators	<code>znconreylog(G, m)</code>
conductor of χ (χ_0 primitive)	<code>znconreyconductor($G, \chi, \{\chi_0\}$)</code>

True-False Tests

is x the disc. of a quadratic field?	<code>isfundamental(x)</code>
is x a prime?	<code>isprime(x)</code>
is x a strong pseudo-prime?	<code>ispseudoprime(x)</code>
is x square-free?	<code>issquarefree(x)</code>
is x a square?	<code>issquare($x, \{\&n\}$)</code>
is x a perfect power?	<code>ispower($x, \{k\}, \{\&n\}$)</code>
is x a perfect power of a prime? ($x = p^n$)	<code>isprimepower($x, \&n\}$)</code>
... of a pseudoprime?	<code>ispseudoprimepower($x, \&n\}$)</code>
is x powerful?	<code>ispowerful(x)</code>
is x a totient? ($x = \varphi(n)$)	<code>istotient($x, \{\&n\}$)</code>
is x a polygonal number? ($x = P(s, n)$)	<code>ispolygonal($x, s, \{\&n\}$)</code>
is pol irreducible?	<code>polisirreducible(pol)</code>

Graphic Functions

crude graph of $expr$ between a and b	<code>plot($X = a, b, expr$)</code>
High-resolution plot (immediate plot)	
plot $expr$ between a and b	<code>plotth($X = a, b, expr, \{flag\}, \{n\}$)</code>
plot points given by lists lx, ly	<code>plotthraw($lx, ly, \{flag\}$)</code>
terminal dimensions	<code>plotsizes()</code>

Rectwindow functions

init window w , with size x, y	<code>plotinit(w, x, y)</code>
erase window w	<code>plotkill(w)</code>
copy w to w_2 with offset (dx, dy)	<code>plotcopy(w, w_2, dx, dy)</code>
clips contents of w	<code>plotclip(w)</code>
scale coordinates in w	<code>plotscale(w, x_1, x_2, y_1, y_2)</code>
<code>plot</code> in w	<code>plotrecth($w, X = a, b, expr, \{flag\}, \{n\}$)</code>
<code>plot</code> throw in w	<code>plotrecththrow($w, data, \{flag\}$)</code>
draw window w_1 at $(x_1, y_1), \dots$	<code>plotdraw($[[w_1, x_1, y_1], \dots]$)</code>

Low-level Rectwindow Functions

set current drawing color in w to c	<code>plotcolor(w, c)</code>
current position of cursor in w	<code>plotcursor(w)</code>
write s at cursor's position	<code>plotstring(w, s)</code>
move cursor to (x, y)	<code>plotmove(w, x, y)</code>
move cursor to $(x + dx, y + dy)$	<code>plotrmove(w, dx, dy)</code>
draw a box to (x_2, y_2)	<code>plotbox(w, x_2, y_2)</code>
draw a box to $(x + dx, y + dy)$	<code>plotrbox(w, dx, dy)</code>
draw polygon	<code>plotlines($w, lx, ly, \{flag\}$)</code>
draw points	<code>plotpoints(w, lx, ly)</code>
draw line to $(x + dx, y + dy)$	<code>plotrline(w, dx, dy)</code>
draw point $(x + dx, y + dy)$	<code>plotrpoint(w, dx, dy)</code>
draw point $(x + dx, y + dy)$	<code>plotrpoint(w, dx, dy)</code>

Convert to Postscript or Scalable Vector Graphics

The format f is either "ps" or "svg".	
as <code>plot</code>	<code>plotlexport($f, X = a, b, expr, \{flag\}, \{n\}$)</code>
as <code>plot</code> throw	<code>plotthrowlexport($f, lx, ly, \{flag\}$)</code>
as <code>plot</code> draw	<code>plotlexport($f, [[w_1, x_1, y_1], \dots]$)</code>

Based on an earlier version by Joseph H. Silverman
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